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 DRAFT

# Spontaneous $Z_2$ Symmetry Breaking in the Orbifold Daughter of $\mathcal{N} = 1$ Super-Yang–Mills Theory, Fractional Domain Walls and Vacuum Structure

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## Abstract

We discuss the fate of the  $Z_2$  symmetry and the vacuum structure in an  $SU(N) \times SU(N)$  gauge theory with one bifundamental Dirac fermion. This theory can be obtained from  $SU(2N)$  supersymmetric Yang–Mills (SYM) theory by virtue of  $Z_2$  orbifolding. We analyze dynamics of domain walls and argue that the  $Z_2$  symmetry is spontaneously broken. Since unbroken  $Z_2$  is a necessary condition for non-perturbative planar equivalence we conclude that the orbifold daughter is nonperturbatively nonequivalent to its supersymmetric parent. *En route*, our investigation reveals the existence of *fractional* domain walls, similar to fractional D-branes of string theory on orbifolds. We conjecture on the fate of these domain walls in the true solution of the  $Z_2$ -broken orbifold theory. We also comment on relation with nonsupersymmetric string theories and closed-string tachyon condensation.

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# 1 Introduction

Recently, a considerable progress has been achieved [1, 2, 3] in understanding of nonsupersymmetric Yang–Mills theories which can be obtained from supersymmetric gluodynamics by orbifolding or orientifolding, following the original discovery of planar equivalence [4, 5, 6, 7, 8, 9, 10]. While establishing perturbative planar equivalence is quite straightforward, the issue of nonperturbative equivalence of the orbi/orientifold daughters to the parent theory — supersymmetric gluodynamics — is more complicated. The question of nonperturbative equivalence between supersymmetric (SUSY) and non-SUSY theories was raised by Strassler [11] who formulated a *nonperturbative orbifold conjecture* (NPO). Shortly after Strassler’s work, arguments were given [12, 13] that in the orbifold daughter planar equivalence fails at the nonperturbative level. In particular, Tong showed that when the orbifold theory is compactified on a spatial circle, the SYM-inherited vacuum is not the genuine vacuum of the theory [13]. It was discovered, however, that the orientifold daughter is more robust and withstands the passage to the nonperturbative level [1, 2, 3].

A refined proof of the nonperturbative equivalence of the orientifold daughter was worked out in Ref. [3]. Here we carry out a similar analysis for the orbifold theory. This analysis will show, in a very transparent manner, that the necessary condition for the nonperturbative equivalence to hold in the orbifold case is that the  $Z_2$  symmetry of the  $(Z_2)$  orbifold Lagrangian is not spontaneously broken. The same conclusion was reached in [14].

As well-known [15, 16], string theory prompts us that, for the orbifold daughter of  $\mathcal{N} = 4$  SYM theory, the  $Z_2$  symmetry is spontaneously broken above a critical value of the ’t Hooft coupling. The orbifold field theory under consideration can be described by a brane configuration of type-0 string theory [10] (see Sect. 6). Type-0 strings contain a closed-string tachyon mode in the twisted sector. The tachyon couples to the “twisted” field [16]

$$T \equiv (\text{Tr } F_e^2 - \text{Tr } F_m^2) \quad (1)$$

of the  $\text{SU}_e(N) \times \text{SU}_m(N)$  gauge theory.<sup>1</sup> The subscripts  $e$  and  $m$  refer to

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<sup>1</sup>Here and below the normalization of traces is such that

$$\text{Tr } F \tilde{F} = \sum_{a=1}^{4N^2} F_{\mu\nu}^a \tilde{F}^{\mu\nu a}, \quad \text{Tr } (F \tilde{F})_e = \sum_{a=1}^{N^2} \left( F_{\mu\nu}^a \tilde{F}^{\mu\nu a} \right)_e,$$

“electric” and “magnetic”, respectively. The words electric and magnetic, borrowed from the string theory terminology, are used here just to distinguish between the two  $SU(N)$ ’s of the gauge group  $SU(N) \times SU(N)$ .

The prediction of string theory [16] is that the perturbative vacuum at  $\langle T \rangle = 0$  is unstable. In the *bona fide* vacua a condensate of the form

$$\langle \text{Tr } F_e^2 - \text{Tr } F_m^2 \rangle = \pm \Lambda^4 \quad (2)$$

must develop.

Our task is to explore this phenomenon within field theory *per se*, with no (or almost no) reference to string theory. The  $SU_e(N) \times SU_m(N)$  gauge theory with a Dirac bifundamental field is very interesting by itself, with no reference to orbifolding. If we could *prove* that  $Z_2$  is spontaneously broken, using field-theoretic methods, this would be a tantalizing development. Below we will present arguments that such spontaneous symmetry breaking does take place, which, although convincing, stop short of being a full proof.

First, we generalize the analysis of Ref. [3] to demonstrate that NPO does require unbroken  $Z_2$ . Then we proceed to arguments based on consideration of domain wall dynamics to show that the domain wall of the parent SYM theory, upon orbifolding, becomes unstable and splits into two walls: one “electric” and one “magnetic.” As will be explained below, this splitting is a signal of the spontaneous breaking of the  $Z_2$  symmetry.

We then argue that the true solution of the orbifold theory has vacua in which the tachyon operator condenses. We discuss the true vacuum structure of the orbifold theory and comment on its relevance for the issue of the closed-string–tachyon condensation in string theory.

The paper is organized as follows. In Sect. 2 we show that NPO requires unbroken  $Z_2$  symmetry and provide evidence that this symmetry is broken. Section 4 is devoted to a discussion of the order parameter(s) in the orbifold theory. In Sect. 5 we discuss dynamics of fractional domain walls. Section A is devoted to low-energy theorems. Section 6 discusses the relation between type-0 string theory and the  $Z_2$  orbifold field theory. In Sect. 7 we comment on the difference between orbifold and orientifold daughters. Finally, in Sect. 8 we summarize our results and outline possible issues for future investigations.

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and so on.

After the first version of this paper appeared in the eprint form, a related work was submitted [17]. We agree with a part of criticism presented [17]. In particular, in Ref. [12] and in the first version of this paper low-energy theorems were used to discriminate between the parent and orbifold daughter theories. These theorems become instrumental under the assumption of coincidence between the corresponding vacuum condensates. The vacuum condensate coincidence, imposed previously, is seemingly not necessary and, in fact, does not hold in a toy model we have recently analyzed. Relaxing this requirement makes the above low-energy theorems (and gravitational anomalies) uninformative. If one allows for unequal condensates, they cannot be used to prove (or disprove) that the  $Z_2$  symmetry is broken. We revised the manuscript accordingly.

We strongly disagree, however, with the analysis of the domain wall issue presented in [17]. Domain walls dynamics in  $Z_2$  orbifold field theories is incompatible with planar equivalence.

## 2 The role of $Z_2$ in the proof of planar equivalence

Let us analyze whether or not nonperturbative planar equivalence takes place in the orbifold theory, following the line of reasoning established in [3]. We refer the reader to this paper for a detailed discussion of the procedure.

Here we will consider, as a particular example, a two-fermion-loop contribution (see Fig. 1) to the partition function.

Each fermion loop consists of two lines: one solid and one dashed. The solid line denotes propagation of the (fundamental) color index belonging to the “electric”  $SU(N)$  while the dashed line denotes propagation of the color index belonging to the “magnetic”  $SU(N)$ . “Electric” and “magnetic” gluon fields are marked in Fig. 1 by vertical and horizontal shadings, respectively.

Each fermion loop represents, in fact,

$$\Gamma[A] = \log \det (i \not{\partial} + \not{A}^a T^a - m) . \quad (3)$$

A mass term is introduced for regularization. It is very important (see below). In what follows we will assume for definiteness that  $m$  is real and  $m > 0$ . We note that

$$A T = A_e T_e + A_m T_m . \quad (4)$$

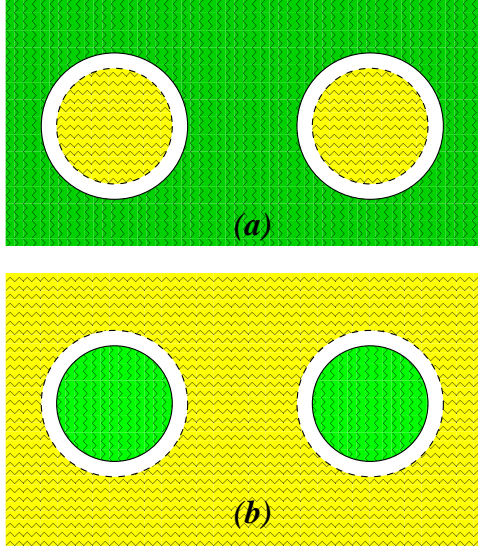


Figure 1: The fermion loop expansion of the partition function at  $N \rightarrow \infty$ .

Moreover,

$$\begin{aligned}
\Gamma_r[A, J_\Psi] &= -\frac{1}{2} \int_0^\infty \frac{dT}{T} \\
&\times \int \mathcal{D}x \mathcal{D}\psi \exp \left\{ - \int_\epsilon^T d\tau \left( \frac{1}{2} \dot{x}^\mu \dot{x}^\mu + \frac{1}{2} \psi^\mu \dot{\psi}^\mu - \frac{1}{2} m^2 \right) \right\} \\
&\times \text{Tr } \mathcal{P} \exp \left\{ i \int_0^T d\tau \left( A_\mu^a \dot{x}^\mu - \frac{1}{2} \psi^\mu F_{\mu\nu}^a \psi^\nu \right) T^a \right\}, \quad (5)
\end{aligned}$$

with the same convention regarding  $AT$  and  $FT$  as in Eq. (4). In the above expressions the gluon field is considered as background. Averaging over the vacuum gluon field is performed at the very end.

The requirement that the fermion loops are connected through the gluon field enforces that only selected contractions are possible in the orbifold theory. In particular on the diagrams of Fig. 1 the outside loops must be *both* either solid lines or dashed lines (*a*, *b*, respectively). Solid-dashed combination is excluded as it represents a disconnected graph. In the parent SYM theory we deal with a single  $SU(2N)$ , and all contractions are possible.

In perturbation theory the contributions from the diagrams *1a* and *1b* are equal. The combinatorics is such that adding up *1a* and *1b* one exactly reproduces the two-fermion-loop contribution in SYM theory provided one performs the following coupling rescaling:

$$g_D^2 = 2 g_P^2, \quad (6)$$

where  $P$  and  $D$  stand for the parent and daughter (orbifold) theories. The above rescaling ensures that the 't Hooft couplings are the same in the parent and daughter theories.

In the perturbative planar equivalence — a solidly established fact — the vacuum angle  $\theta$  plays no role since it does not show up in perturbation theory. A correspondence between parent and daughter  $\theta$ 's following from NPO can be derived from holomorphic dependences of the bifermion condensates on the complexified coupling constants. If the vacuum angles in the parent and orbifold daughter theories are introduced as

$$\begin{aligned} \Delta\mathcal{L}_P &= \frac{\theta_P}{32\pi^2} F_{\mu\nu}^a \tilde{F}^{\mu\nu,a}, \\ \Delta\mathcal{L}_D &= \frac{\theta_D}{32\pi^2} \sum_{\ell=e,m} F_{(\ell)\mu\nu}^a \tilde{F}_{(\ell)}^{\mu\nu,a}, \end{aligned} \quad (7)$$

then it is not difficult to show that the vacuum angles must be rescaled too [18, 12, 14],

$$\theta_D = \frac{1}{2} \theta_P. \quad (8)$$

Equations (6) and (8) are equivalent to the statement of correspondence between the holomorphic coupling constants.

### 3 Planar equivalence: what does it mean?

In establishing large- $N$  equivalence between distinct theories, with distinct vacuum structure (and, as we will see shortly, the vacuum structure of the parent theory is not maintained upon projection to the orbifold daughter theory) we must carefully specify what this equivalence might actually mean. Any theory is characterized by a set of physical quantities that scale differently in the 't Hooft limit. For instance, particle masses are assumed to be

$N$ -independent, their residues in appropriate currents grow with  $N$ , particle widths fall with  $N$  (so that at  $N \rightarrow \infty$  all mesons are stable), the vacuum energy density scales as  $N^2$ , the number of vacua scales as  $N^1$ , and so on. When two theories with distinct vacuum structure are compared (but with the same scale parameter  $\Lambda$ ), physical equivalence of the theories in question need not necessarily mean full general equality of all  $n$ -point functions, since such equality may come into contradiction with appropriate scaling laws. In particular, some vacuum condensates and low-energy theorems can be sensitive to the number of fundamental degrees of freedom (cf. the  $\pi^0 \rightarrow 2\gamma$  constant whose consideration led to the conclusion of three colors in *bona fide* QCD in the early 1970s).

It is clear that a minimal requirement of planar equivalence is coincidence of the particle spectra in the common sector. More precisely, let us consider a vacuum  $V_P$  of the parent theory that can be mapped onto a vacuum  $V_D$  of the daughter theory and vice versa. We must verify that both  $V_P$  and  $V_D$  are stable vacua. If the spectra of particle excitations in both vacua in the common sector coincide up to  $1/N$  corrections we can speak of planar equivalence.

Besides particle excitations the parent and daughter theories may (and do) support extended excitations, such as domains walls. To consider domain walls we must consider pairs of vacua  $V_P, V'_P$  and  $V_D, V'_D$  which can be mutually mapped. Correspondence between the parent and daughter walls can be included in the requirement of planar equivalence.

In the remainder of this section we show that evident distinctions in the vacuum structure of the parent SYM theory and its orbifold daughter lead, with necessity, to a mismatch in certain correlation functions. In particular,  $\theta$  dependences cannot match. This does not necessarily mean a mismatch in the particle spectra since the composite particle masses acquire a dependence on the  $\theta$  parameters introduced in Eqs. (7) (at  $m \neq 0$ , where  $m$  is a small fermion mass term, see below) only in subleading order in  $1/N$ .

The functional-integral representation for the partition function, with the fermion determinant included, is ill-defined unless we regularize the determinant. The infrared regularization is ensured by the introduction of a (small) mass term  $m$ . Simultaneously, this mass term lifts the vacuum degeneracy, eliminating further ambiguities in the functional integral.

Let us dwell on the vacuum structure of the parent and daughter theories.  $SU(2N)$  supersymmetric gluodynamics has  $2N$  vacua labeled by the order



parameter, the gluino condensate,<sup>2</sup>

$$\langle \lambda_\alpha^a \lambda^{a,\alpha} \rangle = -6(2N) \Lambda^3 \exp \left( i \frac{2\pi k + \theta_P}{2N} \right), \quad k = 0, 1, \dots, 2N - 1, \quad (9)$$

Its  $SU(N) \times SU(N)$  orbifold daughter has  $N$  vacua (under the assumption that  $Z_2$  is unbroken) which are labeled by the order parameter

$$\left\langle \frac{1}{2} \bar{\Psi} (1 - \gamma_5) \Psi \right\rangle \sim N \Lambda^3 \exp \left( i \frac{2\pi k + \theta_D}{N} \right), \quad k = 0, 1, \dots, N - 1, \quad (10)$$

Consider an instructive example, namely,

$$\theta_D = \pi, \quad \theta_P = 2\pi, \quad (11)$$

and  $m$  real and positive, the vacuum structure is depicted in Fig. 2.  $P_0$  is the unique vacuum of the SYM theory, while  $D_{\pm 1}$  are the vacua of the orbifold theory. Note that at  $m > 0$  the “vacua”  $P_{\pm 1}$  are in fact excited (or quasistable) because

$$\mathcal{E}_{P_{\pm 1}} > \mathcal{E}_{P_0}.$$

The daughter theory has two-fold degeneracy,

$$\mathcal{E}_{D_{+1}} = \mathcal{E}_{D_{-1}},$$

a phenomenon well-known at  $\theta = \pi$ . This is the so-called Dashen phenomenon [24], with all ensuing consequences. Let us emphasize that physics at  $\theta = \pi$  and  $\theta = 0$  are essentially different. In particular, at  $\theta = \pi$  spontaneous breaking of discrete symmetries (such as  $P$ -invariance) typically occurs [24].

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<sup>2</sup>The gluino condensate in supersymmetric gluodynamics was first conjectured, on the basis of the value of his index, by E. Witten [19]. It was confirmed in an effective Lagrangian approach by G. Veneziano and S. Yankielowicz [20], and exactly calculated by M. A. Shifman and A. I. Vainshtein [21]. The exact value of the coefficient  $-12N$  in Eq. (9) (for  $SU(2N)$ ) can be extracted from several sources. All numerical factors are carefully collected for  $SU(2)$  in the review paper [22]. A weak-coupling calculation for  $SU(N)$  with arbitrary  $N$  was carried out in [23]. Note, however, that an unconventional definition of the scale parameter  $\Lambda$  is used in Ref. [23]. One can pass to the conventional definition of  $\Lambda$  either by normalizing the result to the  $SU(2)$  case [22] or by analyzing the context of Ref. [23]. Both methods give one and the same result.

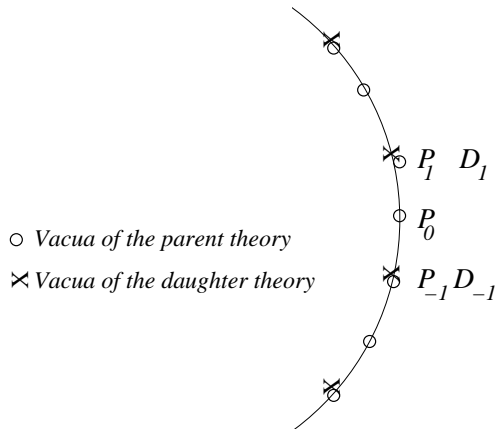


Figure 2: The vacuum structure in the  $SU(2N)$  SYM theory and its  $SU(N) \times SU(N)$  orbifold daughter on the complex plane of the order parameter (the bifermion condensate  $-\langle \lambda_\alpha^a \lambda^{a,\alpha} \rangle$  or  $-\langle \bar{\Psi} (1 - \gamma_5) \Psi \rangle$ , respectively).

One can consider another instructive example,  $\theta_P = \pi$ . At this point, the Dashen phenomenon occurs in the parent theory. There is a double-fold vacuum degeneracy. At  $m \neq 0$  domain walls are unstable, generally speaking. However, a stable domain walls emerges in the Dashen point, as is well-known from the past. At the same time, the corresponding vacuum angle of the daughter theory at this point is  $\theta_D = \pi/2$ . The Dashen point is not yet reached. It is clear that there is no equivalence in this aspect.

Coincidence of the vacuum structure in the parent and daughter theories at  $N = \infty$  implies, generally speaking, a much broader understanding of planar equivalence. This is the case for orientifold daughters. For orbifold daughters one has to stick to the minimal requirement specified in the beginning of this section.

## 4 Order parameters

We will pause here to discuss appropriate order parameters. In the parent SYM theory the order parameter is the gluino condensate (9). In the daughter theory with the spontaneously broken  $Z_2$  the bifermion condensate (10) is insufficient for differentiation of all  $2N$  vacua of the theory because it is  $Z_2$ -even. We must supplement it by a  $Z_2$ -odd expectation value of (1). This vacuum expectation value (VEV) is dichotomic. The bifermion condensate (10) in conjunction with  $\langle T \rangle = \pm \Lambda^4$  fully identifies each of the  $2N$  degenerate vacua of the orbifold theory. Somewhat symbolically the vacuum structure is presented in Fig. 3. The angular coordinate represents the phase of (10), while the radial coordinate can take two distinct values representing the dichotomic parameter  $\langle T \rangle$ .

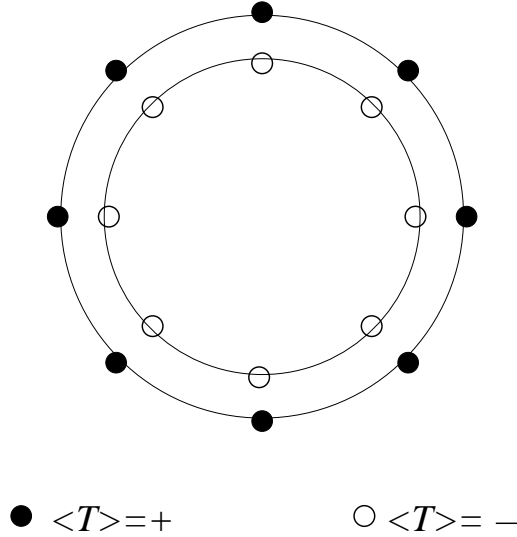


Figure 3: The vacuum structure of the  $SU(8) \times SU(8)$  orbifold theory.

It is instructive to discuss here the  $Z_2$ -even gluon condensate  $\langle F_e^2 + F_m^2 \rangle$ . This operator is related to the total energy-momentum tensor of the theory,

$$\theta_\mu^\mu = -\frac{3N}{32\pi^2} \sum_{\ell=e,m} (F_{\mu\nu}^a F_{\mu\nu}^a)_\ell . \quad (12)$$

Since all  $2N$  vacua are degenerate, at first sight the gluon condensate is no order parameter, since the VEV of (12) is the same in all vacua. Ever since the gluon condensate was introduced in non-Abelian gauge theories [25] people tried to identify it as an order parameter. In a sense, in the case at hand it is!

To be more precise, a nonvanishing (in the *planar approximation*) VEV  $\langle F_e^2 + F_m^2 \rangle \neq 0$  signals the spontaneous breaking of  $Z_2$ . Indeed, if  $Z_2$  is unbroken  $\langle F_e^2 + F_m^2 \rangle$  reduces to  $\langle F_{\text{SYM}}^2 \rangle$ . The latter condensate vanishes due to supersymmetry of the parent theory. Hence, the  $Z_2$  symmetric vacua in the daughter theory would have vanishing vacuum energy density. Since the  $Z_2$ -symmetric point is unstable, the *bona fide*  $Z_2$ -asymmetric vacua must have a negative energy density. Equation (12) implies then that in the genuine  $Z_2$ -broken vacua

$$\langle F_e^2 + F_m^2 \rangle \neq 0. \quad (13)$$

Thus, in the case at hand the gluon condensate does play the role of an order parameter, much in the same way as  $\langle F_{\text{SYM}}^2 \rangle$  is the order parameter for SUSY breaking in SUSY gluodynamics. Note that for this reason  $\langle F_e^2 + F_m^2 \rangle$  must vanish in (planar) perturbation theory. *Nonperturbatively*,<sup>3</sup> Eq. (13) must hold at  $O(N^2)$ . This prediction from the broken  $Z_2$  symmetry imposes a strong restriction on the low-energy effective action for the orbifold daughter. In particular it disfavors the action suggested in [27].

Needless to say, revealing dynamical distinctions leading to vanishing/non-vanishing of  $\langle F^2 \rangle$  in the parent/daughter theory is of paramount importance. We are far from understanding these mechanisms. We would like to make a single remark regarding instantons, the only well-studied explicit examples of nonperturbative field configurations. In the SYM theory instanton does not contribute to the vacuum energy because of the fermion zero modes (an instanton-antiinstanton configuration could contribute but it is topologically unstable.) The orbifold theory exhibits a new phenomenon (to the best of our knowledge, for the first time ever): topologically *stable instanton-antiinstanton pair*, connected through fermion zero modes, see Fig. 4. The stability is due to the fact that they belong to distinct gauge factors. Therefore, although the overall topological charge vanishes (all fermion zero modes

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<sup>3</sup>In the orientifold theory the gluon condensate vanishes at the leading order. A non-vanishing gluon condensate at subleading order was detected in the orientifold theory in Ref. [26].

are contracted), still  $\text{instanton}_e$  cannot annihilate  $\text{antiinstanton}_m$ .

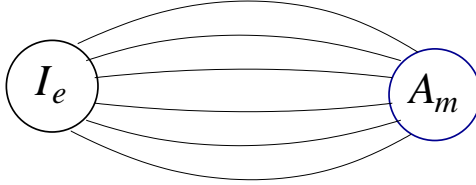


Figure 4: Topologically stable instanton-antiinstanton pair in the orbifold theory. Instanton belongs to the electric  $SU(N)$  while antiinstanton to the magnetic  $SU(N)$ .

## 5 Domain wall dynamics in orbifold field theory

In this section we discuss the dynamics of domain walls in the  $Z_2$  orbifold field theory. We discuss both four-dimensional and world-volume dynamics. Since domain walls are “QCD D-branes” [28] the similarity between wall dynamics and D-brane dynamics is clear. In Sect. 6 we will discuss the dynamics of D-branes in type-0 string theory. We will identify the domain walls of the orbifold daughter theory with the fractional D-branes of type-0 string theory.

Why domain walls? As well-known, the occurrence of domain walls is the physical manifestation of spontaneously broken discrete symmetries. Since our considerations aim at exploring the  $Z_2$  breaking in the orbifold daughter theory, an analysis of the domain walls is relevant.

In addition, we will discuss the role of the fractional domain walls of the orbifold theory as fundamental (or constituent) domain walls of the theory in its true vacuum.

### 5.1 Four-dimensional perspective

Let us consider the domain walls in the  $Z_2$  orbifold field theory. It is a  $SU_e(N) \times SU_m(N)$  gauge theory with a bifundamental Dirac fermion. The theory has a global  $U_A(1)$  axial anomaly analogous to the  $U_{\mathcal{R}}(1)$  anomaly in the parent SYM theory. On the basis of the  $U_A(1)$  anomaly one can deduce that the daughter theory has  $N$  degenerate vacua marked by distinct values of the bifermion condensate  $\langle \bar{\Psi}(1 - \gamma_5)\Psi \rangle$  (see Fig. 2). The domain walls

can separate these  $N$  vacua. (An alternative terminology: the domain walls can interpolate between these  $N$  vacua.) Let us begin with a brief review of the SYM theory domain walls.

The SYM theory contains BPS domain walls [29] that carry both tension  $\sigma$  and charge  $Q$  (per unit area), with  $\sigma = Q$ . The expressions for the tension and charge are [30]

$$\sigma = \frac{3(2N)}{32\pi^2} \int_{\text{wall}} dz \text{Tr } F^2, \quad (14)$$

$$Q = \frac{3(2N)}{32\pi^2} \int_{\text{wall}} dz \text{Tr } F \tilde{F}, \quad (15)$$

where  $z$  is the direction perpendicular to the wall plane. Equation (14) is a consequence of the scale anomaly.

We can consider, as well, the bound state of  $k$  elementary walls. These walls interpolate between the vacua  $i$  and  $i + k$ . The *exact* tension for the  $k$ -wall configuration is [29]

$$\sigma(k) = \Lambda^3 (2N)^2 \sin \frac{\pi k}{2N}. \quad (16)$$

At  $N \rightarrow \infty$  it reduces to

$$\sigma(k) \rightarrow k\sigma(1). \quad (17)$$

In other words, the walls do not interact as their total tension is the sum of tensions of  $k$  *free* 1-walls. Although the walls do interact via the exchange of glueballs, there is a perfect cancellation between the contribution of even- and odd-parity glueballs [30]. In Sect. 5.2 we will see, from the world-volume theory standpoint, that the no-force result is due to bose-fermi degeneracy on the wall.

Now we proceed to the orbifold daughter. Analogously to the parent SYM theory, the domain walls of the daughter theory carry both tension and charge which can be evaluated by using the orbifold procedure.

We obtain the following expressions for the tension and charge of the orbifold theory domain walls:<sup>4</sup>

$$\sigma_D = \frac{3N}{32\pi^2} \int_{\text{wall}} dz \text{Tr } F_e^2 + \frac{3N}{32\pi^2} \int_{\text{wall}} dz \text{Tr } F_m^2, \quad (18)$$

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<sup>4</sup>In SYM theory such integrals are well-defined since  $\langle F^2 \rangle$  vanishes in SUSY vacua. In the orbifold theory this is not the case, see Sect. 4. Therefore, the integrals in Eq. (18) require a proper regularization.

$$Q_D = \frac{3N}{32\pi^2} \int_{\text{wall}} dz \text{Tr} (F\tilde{F})_e + \frac{3N}{32\pi^2} \int_{\text{wall}} dz \text{Tr} (F\tilde{F})_m. \quad (19)$$

In a bid to reveal inconsistencies of the NPO conjecture and preparing for such a demonstration in Sect. 5.2, we will look at the domain walls from a slightly different angle. It is suggestive to think of the domain walls of the orbifold theory as of marginally bound states of *fractional* “electric” and “magnetic” domain walls, with the following tensions and charges:

$$\begin{aligned} \sigma_e &= \frac{3N}{32\pi^2} \int dz \text{Tr} F_e^2, & \sigma_m &= \frac{3N}{32\pi^2} \int dz \text{Tr} F_m^2, \\ Q_e &= \frac{3N}{32\pi^2} \int dz \text{Tr} (F\tilde{F})_e, & Q_m &= \frac{3N}{32\pi^2} \int dz \text{Tr} (F\tilde{F})_m. \end{aligned} \quad (20)$$

Assuming now that NPO is valid, *and* the  $Z_2$  symmetry is unbroken, i.e.  $\sigma_e = \sigma_m$ , we get

$$\sigma_{e,m} = \frac{1}{2} (\sigma_e + \sigma_m), \quad (21)$$

i.e., a *fractional* amount of tension (in full analogy with the fractional D-branes, see Sect. 6). The tensions of the fractional multi-walls are

$$\sigma_{e,m}(k) = \frac{1}{2} \Lambda^3 N^2 \sin \frac{\pi k}{N} \rightarrow k \sigma_{e,m}(1) \quad \text{at} \quad N \rightarrow \infty. \quad (22)$$

When  $k = 2$ , the statement reduces to that two parallel electric domain walls do not interact at  $N = \infty$ . Needless to say, the same is valid for the magnetic walls. In Sect. 5.2 we will demonstrate, using the world-volume description, that two electric domain walls do interact at  $N \rightarrow \infty$ .

## 5.2 World-volume (2+1)-dimensional perspective

Our discussion of the world-volume domain wall dynamics in the orbifold daughter is closely related to the situation in the parent SUSY theory. The world-volume theory for  $k$ -walls in  $\mathcal{N} = 1$  gluodynamics was derived in Ref. [31]. It was shown to be a (2+1)-dimensional  $U(k)$  theory with level- $2N$  Chern–Simons term (for the bulk gauge group  $SU(2N)$ ). The world-volume theory has (2+1)-dimensional  $\mathcal{N} = 1$  supersymmetry. Note that  $\mathcal{N} = 1$  SUSY in three-dimensional  $SU(N)$  theory is dynamically broken [32] at small

values of the coefficient in front of the Chern-Simons term,  $k_{cs} \leq N/2$ . However this SUSY breakdown does not happen on the world-volume of multiple domain walls in the parent theory since in this case  $k_{cs} = 2N$ , and gauge group is at most  $SU(N)$ .

The action of the theory is

$$S = \int d^3x \left\{ \text{Tr} \left( -\frac{1}{4e^2} F^2 + \frac{2N}{16\pi} \epsilon^{ijk} A^i F^{jk} + \frac{1}{2} (D_i \Phi)^2 \right) + \text{fermions} \right\} . \quad (23)$$

All fields in the action, including the fermion fields, transform in the adjoint representation of  $U(k)$ . For definiteness, we will consider the case  $k = 2$ , which is in a sense minimal, see Sect. 5.3.

Now, consider the orbifold daughter theory. The world-volume theory becomes, by virtue of the orbifold procedure, a  $U_e(1) \times U_m(1)$  gauge theory with a neutral scalar field and *bifundamental* fermions<sup>5</sup>

$$\begin{aligned} S = & \int d^3x \left\{ \sum_{\ell=e,m} \left( -\frac{1}{4e^2} F_\ell^2 + \frac{N}{16\pi} \epsilon^{ijk} A_\ell^i F_\ell^{jk} + \frac{1}{2} (\partial_i \Phi_\ell)^2 \right) \right. \\ & \left. + \bar{\Psi} (\Phi_e - \Phi_m) \Psi + \dots \right\} . \end{aligned} \quad (24)$$

As we will see momentarily, the occurrence of the Yukawa coupling

$$\bar{\Psi} (\Phi_e - \Phi_m) \Psi \quad (25)$$

in the daughter theory (with no counterpart in the parent one) is a fact of special importance.

We can give the following interpretation to the above expression. The daughter wall consists of a sum of electric and magnetic walls that interact with each other via the bifundamental fermions. In fact, the electric branes can be separated from the magnetic branes. To see that this is the case, note that the Yukawa term (25) in the action (24) can make the bifundamental fermion massive.

Indeed, by giving vacuum expectation values

$$\langle \Phi_e \rangle = v_e , \quad \langle \Phi_m \rangle = v_m , \quad (26)$$

---

<sup>5</sup>The same conclusion about the precise form of the world-volume action can be reached by consideration similar to [31] for type-0 string theory, see Sect. 6.



we generate a mass  $\mu$  for the world-volume fermions,

$$\mu = v_e - v_m. \quad (27)$$

When  $\mu \rightarrow \infty$  the fermions decouple, and we have two decoupled U(1) theories. The interpretation is clear: we can give VEV's and *separate* the electric domain wall from the magnetic one. The world-volume theory on the separated electric (or magnetic) domain walls is just a bosonic U(1) gauge theory with a level- $N$  Chern–Simons term. It is not supersymmetric. There is no reason for the wall tension non-renormalization and the no-force statement.

Let us discuss the force between the two walls. It is done by evaluating the Coleman–Weinberg potential in the presence of a VEV  $v$ . A similar calculation was performed in Refs. [33, 34]. The result is

$$V/A \sim \int d^3k \ln(k^2 + e^2 v^2) \sim c_0 \Lambda^3 + c_1 \Lambda e^2 v^2 - c_2 \frac{e^4 v^4}{m} + \dots, \quad (28)$$

where  $c_0, c_1$  and  $c_2$  are positive coefficients (independent of  $N$ ),  $\Lambda$  is a UV cut-off and  $m$  is an IR cut-off (the gauge boson Chern–Simons mass). We can set  $c_0$  and  $c_1$  to zero by a fine-tuned renormalization. However, even after renormalization a repulsive  $v^4$  term remains. This is not surprising. A necessary condition for the zero force is a degeneracy between bosons and fermions in the world-volume theory. This is achieved in the parent theory, where the world-volume theory is  $\mathcal{N} = 1$  supersymmetric. However, since the theory on the electric walls of the daughter theory is purely bosonic we found a repulsion. This is in contradiction with the NPO conjecture.

At the end of Sect. 5.1 we assumed NPO and we reached the conclusion that there is no force between two parallel electric walls at  $N \rightarrow \infty$ . However, the microscopic calculation reveals a different answer. Again, the conclusion is that the  $Z_2$  symmetry must be broken.

### 5.3 The fate of electric and magnetic fractional walls as independent constituents in the true solution

By studying the fractional domain-wall dynamics we arrived at the conclusion that the  $Z_2$  symmetry is dynamically broken. Moreover, the gauge theory has

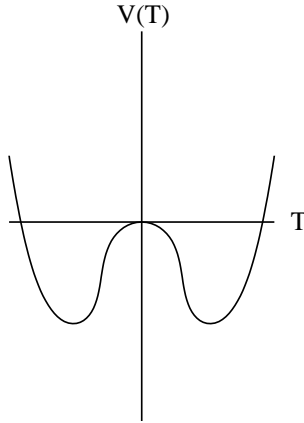


Figure 5: The tachyon field potential. The  $Z_2$  symmetry is dynamically broken in the true vacuum.

$Z_2$  -odd vacua. In other words, the tachyon field potential has a minimum (the tachyon field is  $T = \text{Tr } F_e^2 - \text{Tr } F_m^2$ ).

The statement that  $V(T)$  is bounded from below is not an assumption — it can be justified by observing that the regime of large VEV’s is fully controlled by semiclassical dynamics. From the field-theoretic standpoint it is clear that the only possibility open is that in the *bona fide* vacuum  $\langle T \rangle \sim \Lambda^4$ . At the same time, non-stabilization of tachyons would mean  $\langle T \rangle \gg \Lambda^4$ , which is ruled out. Therefore, the tachyon field potential must look like a Higgs potential, see Fig. 5.

In the parent  $\mathcal{N} = 1$  SYM theory with the gauge group  $SU(2N)$ , there are  $2N$  vacua, with the gaugino condensate as an order parameter. The  $2N$  vacua, being roots of the unity, can be drawn as points on a unit circle, see Fig. 2. The domain walls interpolate between the various vacua.

In the daughter theory the situation is more complicated. Since each vacuum of the  $N$  “false” perturbative vacua splits into two, the vacuum structure of the gauge theory can be described as *two* circles, with  $N$  points on each circle, see Fig. 3.

The wall inheritance from the parent to the daughter theory proceeds as follows. We first pretend that the daughter theory is planar-equivalent to SYM, and that the  $Z_2$  symmetry is unbroken. Then we must start from a 2-wall in the parent theory; and it will be inherited, as the *minimal* wall in the daughter theory. Indeed, if  $Z_2$  is unbroken there are only  $N$  vacua

in the orbi-daughter (versus  $2N$  in SYM). This is seen from Fig. 2. If the wall is inherited, the vacua between which it interpolates must be inherited too. Under NPO only every second vacuum is inherited. Thus, if we want to consider the wall that is inherited, we must consider e.g. the wall connecting  $D_{-1}$  and  $D_1$  in the daughter (this is a minimal wall in the daughter), versus the wall connecting  $P_{-1}$  and  $P_1$  in the parent (this is a 2-wall in SYM).

In the parent theory two 1-walls comprising the 2-wall do not interact with each other (at  $N = \infty$ ). If we consider them on top of each other, the world-volume theory has  $U(2)$  gauge symmetry. However, nobody precludes us from introducing a separation. Then we will have  $U(1)$  on each 1-wall,  $U(1) \times U(1)$  altogether. The tension of each 1-wall is  $1/2$  of the tension of the 2-wall, it is well-defined and receives no quantum corrections. The fact that the world-volume theory on each 1-wall is supersymmetric is in one-to-one correspondence with the absence of quantum corrections.

Now, in the daughter theory, according to NPO, everything should be the same. The minimal wall splits into one electric and one magnetic (the electric one connects  $D_{-1}$  with the would-be vacuum which is a counterpart of  $P_0$ , the magnetic one connects the would-be vacuum which is a counterpart of  $P_0$  with  $D_1$ , each having  $1/2$  of the tension).

However, now the world-volume theories on e-wall and m-wall are not supersymmetric, so that there is no reason for the wall tension non-renormalization.

In this false orbifold theory, there is also no place for the “twisted” walls, since in the false orbifold theory there are no black vacua and white vacua of Fig. 3 — supposedly, there is only one per given value of

$$\left\langle \bar{\Psi} \frac{1 - \gamma_5}{2} \Psi \right\rangle.$$

A possible visualization of the situation is as follows. In the parent theory we have degenerate minima at all points  $P_i$ . In the true orbifold theory these minima become maxima (still critical points, but unstable). Near every second maximum two minima develop. These are true vacua of the true orbifold theory, with  $Z_2$  broken. Of course, the walls that would be inherited from SYM are all unstable, with tachyonic modes. 1-walls are transformed into electric/magnetic walls of the orbifold theory, which are still unstable and, in fact, decay. Each of them separately could decay only into a “twisted wall” connecting white and adjacent black true vacua. The “untwisted”

electric+magnetic wall can decay into a minimal stable wall of the daughter theory which connects two neighboring black vacua or two neighboring white vacua.

## 6 D-branes in type-0B string theory

Orbifold field theories have deep roots in string theory. The particular case of  $Z_2$  orbifold is related to type-0 string. In particular, we can realize the  $Z_2$  orbifold field theory on a brane configuration which involves D4-branes and orthogonal NS5-branes in type-0 string theory [10] (see also [36] for other realizations in type 0B).

Type-0B string theory is a nonsupersymmetric closed string theory, defined by a diagonal Gliozzi–Scherk–Olive (GSO) projection that keeps the following sectors:

$$(NS-, NS-) \oplus (NS+, NS+) \oplus (R+, R+) \oplus (R-, R-). \quad (29)$$

Note the doubling of the R-R fields and the lack of the NS-R sector (closed string fermions). In addition, it is worth noting that the theory contains a tachyon in the (NS-,NS-) sector.

Due to the doubling of the R-R fields the theory contains *two* types of D-branes, often called electric and magnetic branes. A combination of an electric and a magnetic brane is referred to as an untwisted brane. The untwisted brane of the type-0 string is the analogue of the type-II brane. It is useful to think about the electric and magnetic branes as fractional branes, or as the constituents of the untwisted brane.

The field theory on a collection of  $N$  D-branes of type-II string theory is a supersymmetric  $U(N)$  gauge theory. The field theory on a set of  $N$  untwisted D-branes of type-0 string theory is a  $U_e(N) \times U_m(N)$  gauge theory with adjoint scalars and bifundamental fermions [15]. The bosons arise from open strings that connect electric branes with electric branes or magnetic branes with magnetic branes. Fermions are due to open strings that connect electric branes with magnetic branes [37]. The situation is depicted in Fig. 6 below.

Thus, type-0 string theory provides a natural framework for discussing  $Z_2$  orbifold field theories. Indeed, type-0 string theory is a  $Z_2$  orbifold of type-II string theory.

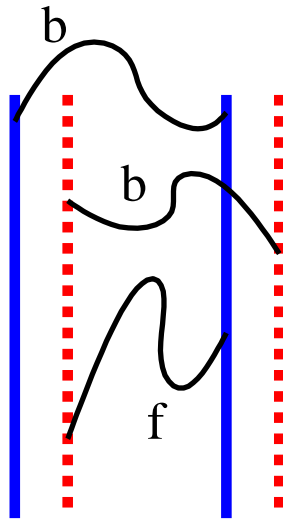


Figure 6: D-branes in type-0 string theory. The open strings that connect electric branes (solid) with electric branes or magnetic branes (dashed) with magnetic branes are bosons. Open strings that connect electric branes with magnetic branes are fermions. The field theory on the brane configuration is a  $U_e(2) \times U_m(2)$  gauge theory with bifundamental fermions.

The forces between D-branes are determined by the annulus diagram. The short-distance force between a set of the same-type branes (electric-electric or magnetic-magnetic) is repulsive [33]. The short-distance force between the opposite brane pair (namely, between electric and magnetic) is attractive. The latter matches the picture we presented in Sect. 5.3. The forces between untwisted branes, namely between a pair of electric plus magnetic branes and another such pair is always zero, as in the supersymmetric theory [34]. The situation is described in Fig. 7 below.

The above results on the forces between the various D-branes of type-0 string theory can be explained via either the closed string channel or the open string channel. Let us start with the closed string channel. In order to achieve a zero force between the branes, the attractive force due to NS-NS modes has to be canceled by the repulsive force due to R-R modes. Note that the cancellation is among bosons of opposite parity; it does not involve fermions (the NS-R sector). No-force situation is achieved in a SUSY set-up (type II) or for dyonic (or untwisted) branes of type 0. However, such a cancellation does not occur in other cases. The same phenomena can be explained via the open string channel. Here, however, the zero force can be explained due to a cancellation between bosons (the NS sector) and fermions (R sector). If the world-volume theory on the brane is SUSY (such as type II) or if the spectrum of the modes on the brane is degenerate, the zero force can be achieved.

## 6.1 D-branes versus domain walls

As has been already mentioned, Witten suggested [28] that domain walls are QCD D-branes. He argued that, since the tension of the domain wall scales as  $N \sim g_{\text{st}}^{-1}$  and since the QCD string can end on the wall, it is natural to conjecture such a relation. Moreover, in [28] Witten described a domain wall as a wrapped M-theory five-brane. Acharya and Vafa later suggested [31] that domain walls correspond to D4-branes wrapping an  $S^2$ . By using this realization Acharya and Vafa were able to determine the world-volume theory.

Following [31] we suggest that the domain walls of the  $Z_2$  orbifold field theory correspond to various branes of type-0 string theory. This is a very natural proposal, since the four-dimensional orbifold field theory itself can be realized on a collection of D-branes of the type-0 string [10].

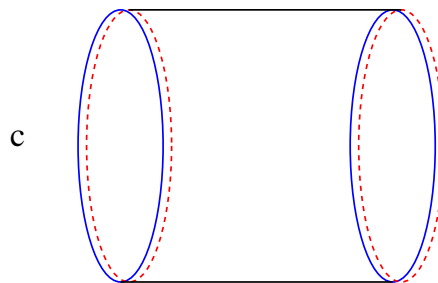
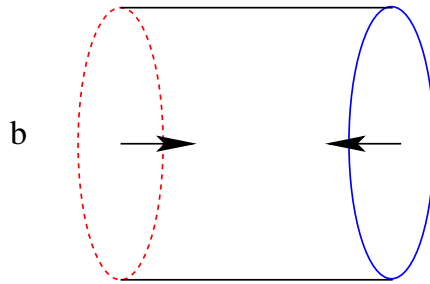
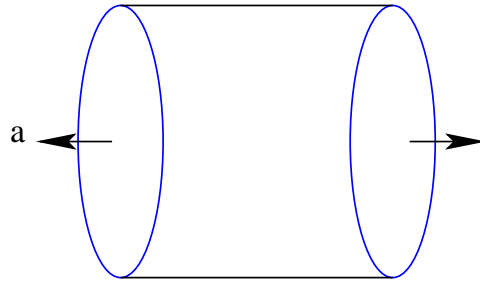


Figure 7: Short distance forces between branes in type-0 string theory: (a) A repulsion between same type branes. (b) An attraction between opposite type branes. (c) There is no force between untwisted branes.

We suggest the following: an electric domain wall corresponds to an electric brane, a magnetic domain wall corresponds to a magnetic brane and, finally, the untwisted domain wall — a pair of electric and magnetic domain walls — corresponds to an untwisted brane.

By using the above identifications and [31] we can get the world-volume theory on various domain walls. The answer follows from an analysis of the annulus diagram in type-0 string theory [37]. For  $k$  coincident electric (or magnetic) domain walls it is a  $(2+1)$ -dimensional  $U(k)$  gauge theory with a real scalar in the adjoint representation and a level- $N$  Chern–Simons term. The theory on the collection of  $k$  untwisted domain walls is a  $U_e(k) \times U_m(k)$  gauge theory with a real scalar in the adjoint representation of each gauge factor, a Chern–Simons term for each factor and bifundamental fermions.

## 6.2 Closed-string tachyon condensation

In the previous sections the breaking of the  $Z_2$  symmetry in the orbifold field theory was proven, mostly within field-theoretical framework, and consequences outlined. An obvious order parameter for the  $Z_2$ -symmetry breaking is the tachyon operator  $T \equiv \text{Tr } F_e^2 - \text{Tr } F_m^2$ , see Eqs. (1), (2). The field-theory analysis suggests that  $T$  acquires a VEV dynamically and develops a potential of the Higgs type (see Fig. 5).

This conclusion is actually very natural, once the relation with type-0 string theory is established. If the orbifold field theory is dual to type-0B string theory on a certain manifold (Maldacena–Nuñez [38], Klebanov–Strassler [39], or  $C^3/Z_2 \times Z_2$  [36]) then by the operator/closed-string relation of the AdS/CFT correspondence, the operator  $\text{Tr } F_e^2 - \text{Tr } F_m^2$  couples to the tachyon mode of the type-0 string [16].

It is also clear that, if there is a duality between a tachyonic string theory and a gauge theory, then the gauge theory must suffer from an instability at strong coupling [16].<sup>6</sup>

The situation, however, is not so simple. The tachyon mode has a negative mass square

$$m^2 = -\frac{2}{\alpha'} \quad (30)$$

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<sup>6</sup>The problems due to the tachyonic mode are solved in orientifold field theories for the simple reason that the dual string theory is nontachyonic, see Sect. 7.



on a flat background, at tree level. The curvature or R-R flux or, maybe, sigma-model corrections can create a potential for the tachyon. It is very difficult to answer the question of the fate of the closed-string tachyon, especially when the theory is compactified on a non-flat manifold and in the presence of the R-R flux.

In the case of the  $Z_2$  daughter of  $\mathcal{N} = 4$  SYM theory, which was conjectured to be dual to type-0B string theory on  $AdS_5 \times S^5$  background, Klebanov and Tseytlin [16] argued that the tachyon mass will be shifted toward positive values, when the 't Hooft coupling is smaller than a certain critical value, namely,

$$\alpha' m^2 = -2 + \frac{c}{\sqrt{\lambda}}. \quad (31)$$

However, the full potential for the tachyon field at *strong* 't Hooft coupling remained unknown. Our field-theory analysis suggests a definite answer to this question. We argue that, if there is a type-0 string model which is dual to the  $Z_2$  orbifold theory, the potential for the tachyon mode is as shown in Fig. 5.

We conclude this section by quoting A. M. Polyakov [40] who discussed the fate of the tachyon of noncritical type-0 string theory in his paper *The Wall of The Cave*:

*Presumably, this tachyon should be of the “good” variety and peacefully condense in the bulk.*

## 7 Orbifolds versus orientifolds

In this short section, we would like to explain the conceptual difference between orbifold field theories and orientifold field theories, or why the conjectured planar equivalence [11] does hold for the orientifold field theories [3] and fails for orbifold ones.

Let us start with orbifold theories. The bold conjecture relates supersymmetric theories with the untwisted sector of the orbifold daughter. It does not address the twisted sector of the gauge theory but *assumes* that the twisted sector is “kosher” (or that it decouples from dynamics of the untwisted sector).

However, as was already argued in the present and previous works, a necessary condition for nonperturbative planar equivalence between a supersymmetric theory and a nonsupersymmetric orbifold daughter, is that the daughter theory inherits the SUSY vacua.

In this paper we demonstrated that a condensate (2) develops, hence the vacuum structure of the orbifold theory is different from that of the parent SUSY theory. Multiple evidence for the condensate (2) was obtained, in particular, via investigation of fractional domain walls.

String theorists are familiar with this phenomenon. Type-II strings on orbifold singularities of the form  $C^3/Z_n$ , or type-0 strings *always* contain a tachyon in the twisted sector (and fractional branes).

For orientifold theories the situation is conceptually different. This non-supersymmetric gauge theory does not contain a twisted sector and, in particular, it does not contain fractional domain walls; hence, it is guaranteed that the theory inherits its vacua from the SUSY parent.

Similarly, the candidate for a string dual of the orientifold theory — Sagnotti’s type-0’ model [41] — does not contain a tachyon since it was projected out by orientifolding.

Thus, either from the string-theory side or from the field-theory side, it is evident that a tachyon-free model is a much better starting point for the investigation of nonsupersymmetric gauge (or string) dynamics.

## 8 Conclusions

The goal of this work is to determine the vacuum structure of a non-supersymmetric gauge theory, the orbifold theory. The problem is extremely difficult, since the answer lies in the nonperturbative regime of the theory. The  $Z_2$  orbifold theory is obtained from  $\mathcal{N} = 1$  SUSY gluodynamics by orbifolding. Nonperturbative planar equivalence for such daughter-parent pairs (SUSY–non-SUSY) was suggested in Ref. [11]. While nonperturbative planar equivalence was proven for orientifold daughter [1, 2, 3], with multiple consequences that ensued almost immediately, theorists continued working on orbifold daughters. Evidence reported in this paper points to nonperturbative nonequivalence. Of course, one can say that on the positive side nonperturbative nonequivalence implies spontaneous breaking of  $Z_2$  of the orbifold daughter.

Our investigation suggests a different picture in the orbifold case. Based on domain wall dynamics we arrived at Fig. 3.  $N$  “pre-vacua” that could be inferred from the chiral condensate (10), due to  $Z_2$  breaking, split into  $2N$  vacua,  $N$  “white” and  $N$  “black.” Each vacuum is uniquely parametrized by two order parameters: the bifermion condensate and the tachyon vacuum expectation value (2). In a theory with multiple vacua, interpolating domain walls of distinct types exist. In the true orbifold solution we have walls connecting two white vacua (or two black ones) which can be interpreted as a bound state of an electric plus magnetic wall pair, each of these e,m-walls being individually unstable. We also have twisted walls interpolating between a black vacuum and a white one.

Several possible directions of future research are at the surface. It would be interesting to investigate other gauge theories applying the set of tools used in the present work. An interesting question is whether there exists at all a daughter non-SUSY orbifold theory whose the vacuum structure *is* inherited from the parent SUSY theory.<sup>7</sup> Another possible line of investigation is a derivation of an effective Lagrangian of the Veneziano–Yankielowicz type [20] that would generalize the Lagrangians [26] and [42] to include effects due to spontaneously broken  $Z_2$ .

Examples of cross-fertilization between string theories and gauge field theories are abundant. In the recent years the direction “from fields to strings” is becoming increasingly useful. The present work suggests another topic along these lines: studying closed-string tachyon condensation basing on the analogous phenomenon in field theory. Detailed analysis of the supergravity solutions corresponding to the strong-coupling limit of the orbifold daughter theory could shed light on these issues.

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<sup>7</sup>A good starting point could be a nonsupersymmetric, tachyon-free, closed-string theory. To the best of our knowledge, the only two examples of non-SUSY tachyon-free string theories are type-0’ and the  $SO(16) \times SO(16)$  heterotic strings. Presumably, they are dual to each other.

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## Appendices

### A Trace anomaly low-energy theorems

Here we will derive and discuss low-energy theorems related to the trace anomaly.

Let the parent theory be  $\mathcal{N} = 1$  SUSY Yang–Mills theory with  $SU(2N)$  gauge group,

$$\mathcal{L} = \frac{1}{g_P^2} \left( -\frac{1}{4} F_{\mu\nu}^a F^{\mu\nu,a} + i \bar{\lambda}_\alpha^a \mathcal{D}^{\dot{\alpha}\alpha} \lambda_\alpha^a \right) + \frac{\theta_P}{32\pi^2} F_{\mu\nu}^a \tilde{F}^{\mu\nu,a}, \quad (\text{A.1})$$

where  $\lambda_\alpha$  is the Weyl spinor in the adjoint. The theory has  $2N$  chiral-asymmetric vacua labeled by the value of the gluino condensate  $\langle \lambda\lambda \rangle$ . We will have to add a small gluino mass term, which will lift the vacuum state from zero and break SUSY, and will make  $\theta$  dependence physical and observable.

The daughter theory is the gauge theory with  $SU(N) \times SU(N)$  gauge group, two Weyl bifundamentals and the rescaling law (6), (8). The Lagrangian of the daughter theory is

$$\mathcal{L} = \frac{1}{g_D^2} \left( -\frac{1}{4} \sum_{\ell=e,m} F_{(\ell)\mu\nu} F_{(\ell)}^{\mu\nu} + i \sum_{\ell=e,m} \bar{\chi}_\ell \mathcal{D} \chi_\ell \right) + \mathcal{L}_\theta, \quad (\text{A.2})$$

where the covariant derivative is defined as  $D_\mu = \partial_\mu - i \sum_\ell A_{(\ell)\mu} T^{(\ell)}$ , while  $T^{(\ell)}$  are the generators of the gauge symmetry with respect to the  $\ell$ -th group  $SU(N)$  (here  $\ell = e, m$ ). The fermion fields have the following color assignment

$$\chi_1 \rightarrow \chi_i^a, \quad \chi_2 \rightarrow \eta_b^j, \quad (\text{A.3})$$

where  $a, b$  are fundamental/antifundamental indices belonging to the first (electric)  $SU(N)$  while  $i, j$  are fundamental/antifundamental indices belonging to the second (magnetic)  $SU(N)$ . Then  $\chi_1 \chi_2 \equiv \chi_i^a \eta_a^i$  is a gauge invariant

chiral order parameter. In the theory (A.2), using the existence of the above parameter, we will introduce the fermion mass term *exactly equal to the projection* of the gluino mass term of the parent theory into the daughter one. The daughter theory is non-supersymmetric.

Now, both the parent theory and its orbifold daughter are endowed with appropriate (small) mass terms for the fermions. The mass terms are needed for (i) IR regularization; (ii) making the vacuum energy density  $\mathcal{E}_{\text{vac}} \sim O(N^2)$ . In the massless limit the  $\mathcal{E}_{\text{vac}} \sim O(N)$ , and it is very hard to track subleading terms. (See, however, Ref. [26].) We will discuss only the terms  $\sim O(N^2)$  in  $\mathcal{E}_{\text{vac}}$ .

Let us use the fact that

$$\theta_\mu^\mu = \frac{\beta_0}{32\pi^2} F_{\mu\nu}^a F_{\mu\nu}^a + \text{ferm mass term} = 4\mathcal{E}_{\text{vac}}; \quad (\text{A.4})$$

$$\beta_0 = -6N \text{ (parent)}; \quad \beta_0 = -3N \text{ (daughter)}. \quad (\text{A.5})$$

Combining it with the fact that  $\mathcal{E}_{\text{vac}} = \text{VEV}$  of the very same mass term, we conclude that

$$-\frac{6N}{2^7\pi^2} \langle F_{\mu\nu}^a F_{\mu\nu}^a \rangle = \frac{3}{4}\mathcal{E}_{\text{vac}} \quad (\text{A.6})$$

in the parent theory, and

$$-\frac{3N}{2^7\pi^2} \left\langle (F_{\mu\nu}^a F_{\mu\nu}^a)_e + (F_{\mu\nu}^a F_{\mu\nu}^a)_m \right\rangle = \frac{3}{4}\mathcal{E}_{\text{vac}} \quad (\text{A.7})$$

in the daughter one, and the vacuum energy density of the parent is twice higher than that of the daughter.

Let us make a comment concerning the order parameters in the daughter theory. From the low-energy theorems [35] we get

$$\begin{aligned} 4 \left\langle (F_{\mu\nu}^a F_{\mu\nu}^a)_e - (F_{\mu\nu}^a F_{\mu\nu}^a)_m \right\rangle &= -\frac{3N}{32\pi^2} \int d^4x \\ &\times \left\{ \langle (F_{\mu\nu}^a F_{\mu\nu}^a)(x), (F_{\mu\nu}^a F_{\mu\nu}^a)(0) \rangle_e - \langle (F_{\mu\nu}^a F_{\mu\nu}^a)(x), (F_{\mu\nu}^a F_{\mu\nu}^a)(0) \rangle_m \right\}, \end{aligned} \quad (\text{A.8})$$

which means that the mixed e-m correlator does not contribute to the condensate of the twisted field. In other words, the condensate of the twisted

field corresponds to a difference in the interactions between the “electric” and “magnetic” domain walls. On the other hand, a similar low-energy theorem for the “untwisted” condensate

$$\begin{aligned}
4 \left\langle (F_{\mu\nu}^a F_{\mu\nu}^a)_e + (F_{\mu\nu}^a F_{\mu\nu}^a)_m \right\rangle &= -\frac{3N}{32\pi^2} \int d^4x \\
&\times \left\{ \left\langle (F_{\mu\nu}^a F_{\mu\nu}^a)_e(x) + (F_{\mu\nu}^a F_{\mu\nu}^a)_m(x), (F_{\mu\nu}^a F_{\mu\nu}^a)_e(0) + (F_{\mu\nu}^a F_{\mu\nu}^a)_m(0) \right\rangle \right\},
\end{aligned} \tag{A.9}$$

shows that the mixed e-m correlator contributes in this case. Let us remark that the mixed instanton-antiinstanton pairs mentioned in Sect. 4 can contribute to the “untwisted” condensate or the vacuum energy only.

## B Topological susceptibilities

We define  $\theta$  terms in Eqs. (7). A few comments on this definition will be presented shortly. It is important that (in the parent theory)

$$\mathcal{E}_P = -\mathcal{E}_{0,P} \cos \frac{\theta_P}{2N}. \tag{B.1}$$

Here  $\mathcal{E}_P$  is the vacuum energy density in the parent theory,  $\mathcal{E}_{0,P}$  is a positive constant (proportional to  $m_{\text{gluino}}\Lambda^3$ ). The  $N$  dependence in Eq. (B.1) follows from Witten-type arguments combined with the fact that there are  $2N$  vacua all entangled in the process of the  $\theta$  evolution in the parent theory. This entanglement leads to apparent periodicity  $2\pi \cdot 2N$  rather than  $2\pi$ .

Differentiating Eq. (B.1) twice with respect to  $\theta_P$ , using Eq. (A.1) and setting  $\theta_P = 0$  after differentiation we get the following result for the topological susceptibility in the parent theory:

$$\mathcal{T}_P \equiv i \int d^4x \left\langle \frac{1}{32\pi^2} F_{\mu\nu}^a \tilde{F}_{\mu\nu}^a(x), \frac{1}{32\pi^2} F_{\mu\nu}^a \tilde{F}_{\mu\nu}^a(0) \right\rangle_{\theta=0} = \mathcal{E}_{0,P} \frac{1}{(2N)^2}. \tag{B.2}$$

Now, let us turn to the daughter theory and discuss the  $\theta$  term in the daughter theory. We introduce  $\theta_D$  in such a way that the physical  $2\pi$  periodicity in  $\theta_D$  is maintained, as indicated in Eq. (7). For convenience we

reproduce the appropriate part here,

$$\mathcal{L}_\theta = \frac{\theta_D}{32\pi^2} \sum_{\ell=e,m} F_{(\ell)\mu\nu} \tilde{F}_{(\ell)}^{\mu\nu}. \quad (\text{B.3})$$

Equation (B.3) is consistent with the physical  $2\pi$  periodicity. Indeed, in the daughter theory one can have instanton just in one of the two  $\text{SU}(N)$ 's, with a trivial background in the other  $\text{SU}(N)$ . Then, the normalization in Eq. (B.3) is standard. It is consistent with Eqs. (6) and (8). This is best seen upon transition to the canonically normalized kinetic terms in both theories. Indeed, then  $g_P^2 \theta_P \leftrightarrow g_D^2 \theta_D$ , and consequently Eq. (6) implies (8).

Next, let us consider the following two-point function

$$\Pi_P(q) = i \int d^4x e^{iqx} \left\langle \frac{1}{32\pi^2} F_{\mu\nu}^a \tilde{F}_{\mu\nu}^a(x), \frac{1}{32\pi^2} F_{\mu\nu}^a \tilde{F}_{\mu\nu}^a(0) \right\rangle_{\theta=0} \quad (\text{B.4})$$

at large  $q$  in the parent theory. At  $q^2 = 0$  it reduces to  $\mathcal{T}_P$ .

Let us normalize its counterpart in the daughter theory in such a way that at large  $q$  (in the perturbative domain) the corresponding planar graphs are *equal*. It is not difficult to see that the equal-normalization condition implies

$$\Pi_D(q) = i \frac{1}{2} \int d^4x e^{iqx} \left\langle \frac{1}{32\pi^2} \sum_{\ell=e,m} F_{\mu\nu}^{a_\ell} \tilde{F}_{\mu\nu}^{a_\ell}(x), \frac{1}{32\pi^2} \sum_{\ell=e,m} F_{\mu\nu}^{a_\ell} \tilde{F}_{\mu\nu}^{a_\ell}(0) \right\rangle_{\theta=0}. \quad (\text{B.5})$$

Indeed, at large  $q^2$  we have

$$\text{Im } \Pi_P(q) = g_P^4 (2N)^2 q^4, \quad \text{Im } \Pi_D(q) = \frac{1}{2} 2 g_D^2 (N)^2 q^4, \quad (\text{B.6})$$

where the factor  $1/2$  in the last term reflects  $1/2$  in the defining equation (B.5), while  $2$  reflects two distinct  $\text{SU}(N)$  gluons in the daughter theory. Given equation (6), the perfect match between  $\Pi_P(q)$  and  $\Pi_D(q)$  at large  $q^2$  is achieved, as we intended. From now on we will drop the subscript  $\theta = 0$  where it is self-evident.

Now, assume the orbifold planar equivalence holds nonperturbatively [11]. Then we must conclude that

$$\Pi_D(q=0) = \Pi_P(q=0), \quad (\text{B.7})$$

implying, in turn, that

$$\begin{aligned} \frac{1}{2} \mathcal{T}_D &\equiv i \frac{1}{2} \int d^4x \left\langle \frac{1}{32\pi^2} \sum_{\ell=e,m} F_{\mu\nu}^{a_\ell} \tilde{F}_{\mu\nu}^{a_\ell}(x), \frac{1}{32\pi^2} \sum_{\ell=e,m} F_{\mu\nu}^{a_\ell} \tilde{F}_{\mu\nu}^{a_\ell}(0) \right\rangle \\ &= \mathcal{T}_P = \frac{\mathcal{E}_{0,P}}{4N^2}. \end{aligned} \quad (\text{B.8})$$

On the other hand, in the daughter theory — remember it has allegedly  $N$  rather than  $2N$  vacua — the dependence of the vacuum energy density on the  $\theta$  angle is as follows:

$$\mathcal{E}_D = -\mathcal{E}_{0,D} \cos \frac{\theta_D}{N}. \quad (\text{B.9})$$

Differentiating twice over  $\theta_D$  and setting  $\theta_D = 0$ , we obtain

$$\mathcal{T}_D = \frac{\mathcal{E}_{0,D}}{N^2}. \quad (\text{B.10})$$

Combining now Eqs. (B.8) and (B.10) we come to the very same conclusion as in Appendix A,

$$\mathcal{E}_{0,P} = 2 \mathcal{E}_{0,D}. \quad (\text{B.11})$$

## C Gravitational chiral anomalies

The parent SUSY gluodynamics and the daughter orbifold theories have classically conserved axial currents which are anomalous at the quantum level. In addition to the gluon anomaly of the Adler-Bell-Jackiw type, one can consider the gravitational anomaly whose existence was first noted in [43, 44].

At first we will have to establish appropriately normalized operators which are related by the orbifold projection. If the axial current in the parent theory (in the Weyl representation) is<sup>8</sup>

$$A_P^\mu = g_P^{-2} \text{Tr} \bar{\lambda}^{\dot{\alpha}} (\sigma^\mu)_{\dot{\alpha}\alpha} \lambda^\alpha, \quad (\text{C.1})$$

its orbifold counterpart is

$$A_D^\mu = g_D^{-2} \bar{\Psi} \gamma^\mu \gamma^5 \Psi, \quad (\text{C.2})$$

---

<sup>8</sup>The trace is normalized in such a way that  $\text{Tr} \bar{\lambda}^{\dot{\alpha}} (\sigma^\mu)_{\dot{\alpha}\alpha} \lambda^\alpha = \bar{\lambda}^{\dot{\alpha} a} (\sigma^\mu)_{\dot{\alpha}\alpha} \lambda^{\alpha a}$ .



where  $\Psi$  is the bifundamental Dirac spinor.

With these definitions the chiral gluon anomaly takes the form

$$\partial_\mu A_P^\mu = \frac{2N}{16\pi^2} \text{Tr} F_{\mu\nu} \tilde{F}^{\mu\nu}, \quad (\text{C.3})$$

$$\partial_\mu A_D^\mu = \frac{N}{16\pi^2} \left\{ \text{Tr} \left( F_{\mu\nu} \tilde{F}^{\mu\nu} \right)_e + \text{Tr} \left( F_{\mu\nu} \tilde{F}^{\mu\nu} \right)_m \right\}. \quad (\text{C.4})$$

Now, let us pass to the gravitational anomalies. For one Dirac fermion it was calculated in [43, 44]

$$\partial_\mu A^\mu = -\frac{1}{192\pi^2} R_{\mu\nu\kappa\lambda} \tilde{R}^{\mu\nu\kappa\lambda}, \quad (\text{C.5})$$

where  $R_{\mu\nu\kappa\lambda}$  is the Riemann tensor. For simplicity we specified Eq. (C.5) to the lowest order in  $h_{\mu\nu}$ . (Otherwise, one must have the covariant derivative on the left-hand side). To the lowest order, the right-hand side is  $O(h_{\mu\nu}^2)$  and

$$\tilde{R}^{\mu\nu\kappa\lambda} = \frac{1}{2} \varepsilon^{\mu\nu\rho\sigma} R_{\rho\sigma}{}^{\kappa\lambda}.$$

Equation (C.5) assumes the axial current normalized to unity, and the unit coupling  $h_{\mu\nu} \theta^{\mu\nu}$ .

Let us examine the gravitational anomalies in the parent and daughter theories, expressing the answer in terms of the right-hand side of (C.5). In the parent theory the coefficient is  $(1/2) \cdot 4N^2 = 2N^2$ , the factor  $1/2$  being associated with the Weyl fermions in Eq. (C.1). In the daughter theory the coefficient is  $N^2$ . This factor is the number of the Dirac degrees of freedom.

## D Additional low-energy theorems in the orbifold theory

The orbifold theory admits a class of low-energy theorems which have no parallel in the parent SYM theory. They seem interesting on their own; some are presented here with a brief comment.

The orbifold theory has two classically conserved currents,

$$V^\mu = \bar{\Psi} \gamma^\mu \Psi, \quad A^\mu = \bar{\Psi} \gamma^\mu \gamma^5 \Psi. \quad (\text{D.1})$$

The axial current  $A^\mu$  is anomalous and can be projected onto its counterpart in the SYM theory. At the same time, the vector current  $V^\mu$  is anomaly free. It has no projection.

We can couple this current  $V^\mu$  to an external gauge boson, a “photon.” Then the orbifold theory becomes an  $SU(N) \times SU(N) \times U(1)$  theory. The  $U(1)$  field strength tensor will be denoted by  $\mathcal{F}_{\mu\nu}$ . Consideration of the scale and chiral anomalies in this theory along the lines suggested in [35] provides us with low-energy theorems for the two-photon couplings to the gluon operators, namely,

$$\left\langle 0 \left| 3N \frac{1}{2^5 \pi^2} \sum_{\ell} (F_{\mu\nu}^a F^{\mu\nu a})_{\ell} \right| 2 \gamma \right\rangle = \frac{4N^2}{3} \frac{1}{2^5 \pi^2} (\mathcal{F}_{\mu\nu} \mathcal{F}^{\mu\nu})_{2\gamma} \quad (\text{D.2})$$

and

$$\left\langle 0 \left| N \frac{1}{2^4 \pi^2} \sum_{\ell} (F_{\mu\nu}^a \tilde{F}^{\mu\nu a})_{\ell} \right| 2 \gamma \right\rangle = -2N^2 \frac{1}{2^4 \pi^2} (\mathcal{F}_{\mu\nu} \tilde{\mathcal{F}}^{\mu\nu})_{2\gamma}, \quad (\text{D.3})$$

where the photons are assumed to be on mass shell and  $(k_1 + k_2)^2 \rightarrow 0$  (here  $k_{1,2}$  are photons’ momenta).

Note that unlike QCD the orbifold theory at hand has no composite Goldstone mesons. Therefore, a subtle point in the derivation of the scale anomaly (D.2) which was revealed in [45] does not show up here.

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